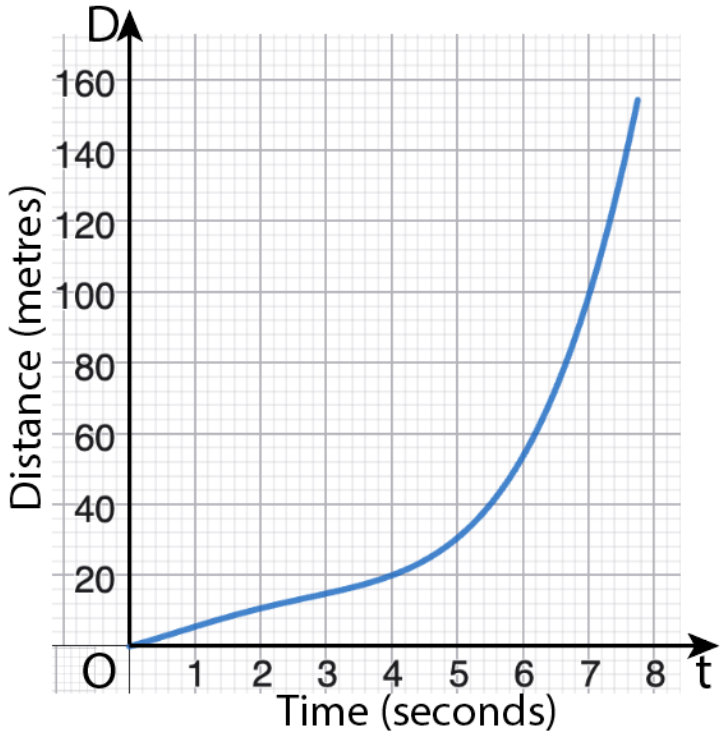
Instantaneous Rates of Change

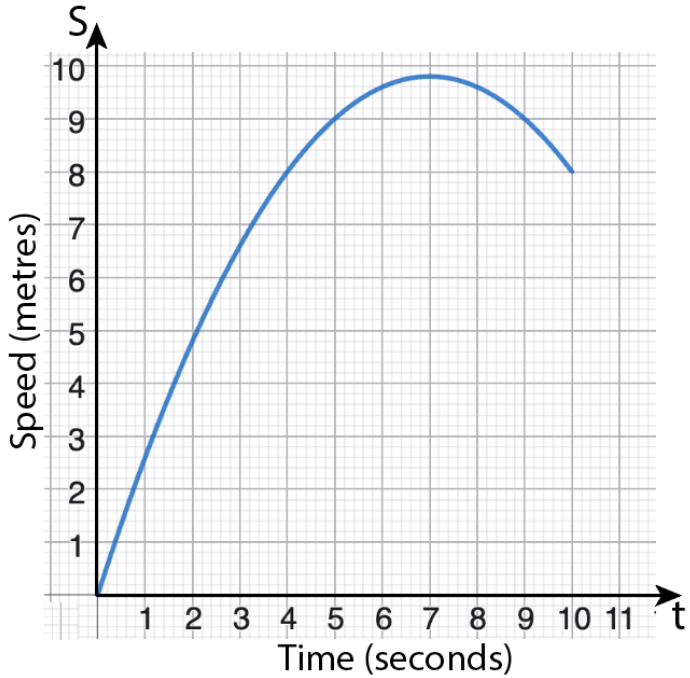
Q1 The distance-time graph shows information about part of a car journey.



Use the graph to estimate the speed of the car at time 5 seconds.

Q2 Elsi runs in a race.

The graph shows her speed, in m/s, t seconds after the start of the race.



a) Calculate an estimate for the gradient of the graph when t = 4

b) Describe fully what your answer to part (a) represents.

c) Explain why your answer to part (a) an estimate is only.

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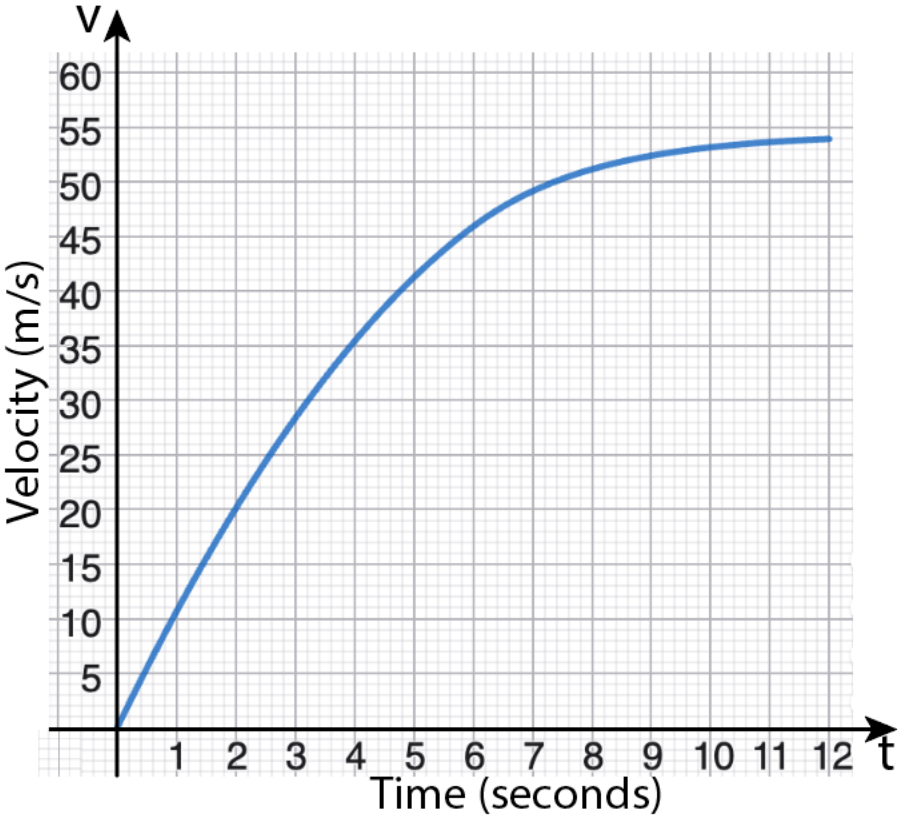
Q3 The graph shows information about the velocity of a parachutist after jumping from a plane.

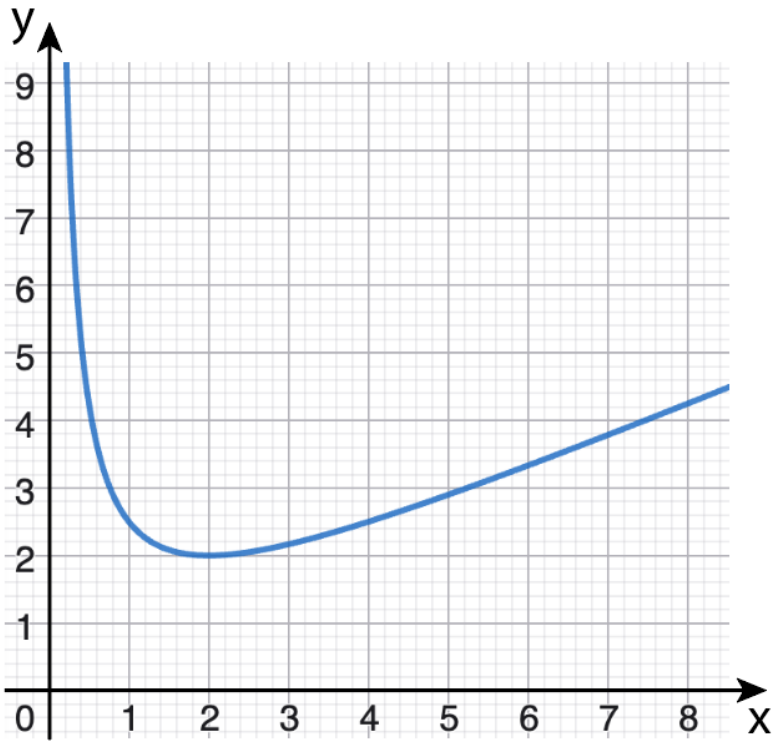


a) Find an estimate of the gradient of the curve after 3 seconds.

b) Interpret the value of the gradient.

Q4 The diagram shows the graph of

$y = \frac{x}{2} + \frac{2}{x}$, for $0 < x \leq 8$.



a) Use the graph to solve the equation $\frac{x}{2} + \frac{2}{x} = 3$.

b) Find an estimate of the gradient of the graph when x = 1.

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Solutions

Q1: Car's speed at t = 5

- At t = 5 the curve is at D ≈ 32 m.
- Draw a tangent at (5, 32). A good tangent passes through roughly (3, 0) and (7, 64).
- Gradient = 64 ÷ 4 = 16.
- Speed ≈ 16 m/s (roughly 14 to 20 m/s is reasonable).

Q2: Elsi's race

a) At t = 4, S ≈ 8. Draw a tangent at (4, 8). It passes through roughly (0, 3.2) and (5, 9.2).
Gradient = 6 ÷ 5 ≈ 1.2 (roughly 0.8 to 1.6 is reasonable).

b) It is Elsi's acceleration 4 seconds after the start. Her speed is increasing by about 1.2 m/s every second at that moment (1.2 m/s²).

c) The tangent is drawn by eye and the coordinates are read from the graph, so neither is exact. The gradient of a curve keeps changing, so a small error in the tangent changes the answer.

Q3: Parachutist

a) At t = 3, v ≈ 29 m/s. Draw a tangent at (3, 29). It passes through roughly (0, 5) and (6, 53).
Gradient = 48 ÷ 6 = 8 (roughly 6 to 10 is reasonable).

b) It is the parachutist's acceleration 3 seconds after jumping. Their velocity is increasing by about 8 m/s every second at that moment (8 m/s²).

Q4: y = x/2 + 2/x

a) Draw the line y = 3 and read off where it crosses the curve: x ≈ 0.8 and x ≈ 5.2.
Check: multiplying through by 2x gives $x^2 - 6x + 4 = 0$, so $x = 3 \pm \sqrt{5}$.
That is 0.76 and 5.24.

b) At x = 1, y = 0.5 + 2 = 2.5. Draw a tangent at (1, 2.5). It passes through roughly (0, 4) and (2, 1).
Gradient = −3 ÷ 2 = −1.5.
Check with calculus: $dy/dx = 1/2 - 2/x^2$, so at x = 1 the gradient is 0.5 − 2 = −1.5 exactly.

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